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Monopolium: the key to monopoles

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Abstract. Dirac showed that the existence of one magnetic pole in the universe could offer an explanation for the discrete nature of the electric charge. Magnetic poles appear naturally in most grand unified theories. Their discovery would be of the greatest importance for particle physics and cosmology. The intense experimental search carried out thus far has not met with success. Moreover, if the monopoles are very massive their production is outside the range of present day facilities. A way out of this impasse would be if the monopoles bind to form monopolium, a monopole–antimonopole bound state, which is so strongly bound that it has a relatively small mass. Under these circumstances it could be produced with present day facilities and the existence of monopoles could be indirectly proven. We study the feasibility of detecting monopolium in present and future accelerators.

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1 Introduction

The theoretical justification for the existence of classical magnetic poles, hereafter called monopoles, is that they add symmetry to Maxwell's equations and explain the quantization of charge [1–3]. Dirac formulated his theory of monopoles considering them basically point-like particles, and quantum mechanical consistency conditions lead to the so called Dirac quantization condition (DQC),

$$eg = \frac{N}{2}\,, \quad N = 1, 2, \dots\,, \tag{1}$$

where e is the electron charge, g the monopole magnetic charge, and we shall use natural units, $\hbar = c = 1$. In this theory the monopole mass, m, is a parameter, limited only by classical reasoning to $m > 2$ GeV [4]. From now on we restrict our discussion to the lowest charge monopole, i.e. $N = 1$ in the Dirac condition (1). In non-Abelian gauge theories monopoles arise as topologically stable solutions through spontaneous breaking via the Kibble mechanism [5–7]. They are allowed by most grand unified theory (GUT) models, have finite size and come out extremely massive $m > 10^{16}$ GeV. There are also models based on other mechanisms with masses between those two extremes [4, 8, 9]. All these monopoles satisfy Dirac's quantization condition as a consequence of their monopole structure and semiclassical quantization [10, 11].

All the attempts to discover or produce monopoles have met with failure [4, 8, 10–14]. This lack of experimental confirmation has led many physicist to abandon hope for their existence. A way out of this impasse is an old idea of Dirac [1, 2, 15], namely, that monopoles are not seen freely because they are confined by their strong magnetic forces forming a bound state called monopolium [16, 17].

Several cosmological scenarios compatible with all cosmological requirements [18–22] have been proposed [16, 23–25] with the aim of making the existence of monopolium compatible with the observation of ultrahigh-energy cosmic rays (UHECRS) [26, 27]. From these cosmological scenarios we have learned that the study of the monopolium annihilation process provides us with information regarding the existence of monopoles, even if they are difficult to detect or to produce in a free asymptotic state. This phenomenon is not a novel feature of physics. Quark–gluon confinement describes the strong limit of quantum chromodynamics, the theory of the hadronic interactions, and their existence is proven by the detection of jets, showers of conventional hadrons. There is, however, a main difference between the two scenarios. In the monopolium case, the elementary constituents may be separated asymptotically, when they are orbiting far from each other, if the energy provided to the system is high enough, while in the

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quark–gluon case this is not possible. In practice, however, there is no big difference, since due to the very high binding energies of monopolium, asymptotic monopoles might only be found, for short periods of time, in the center of galaxies, or clusters of galaxies, not in our laboratories.

In the present work we aim at determining the existence and the dynamics of monopoles in the laboratory. The philosophy behind the present calculation is that the standard model allows for the existence of monopoles, which are spin 0 bosons.[1] Therefore, given the appropriate kinematics we should be able to produce them. However, past experience has taught us that this is not feasible because, most probably, their mass is very large and their production is outside our present experimental capabilities. Our proposal here is that due to the large coupling between monopole and antimonopole the two bind to form a low-mass monopolium state. This state can be produced as an intermediate virtual state and we study its subsequent decays. Thus, in an indirect way monopole physics can be revealed.

2 Monopolium detection

We proceed to discuss signatures of monopolium, the monopole–antimonopole bound state, when produced in e^+e^- annihilation[2]. We use the low-energy effective theory of Ginzburg and Schiller [29, 30] in order to describe the interaction. This theory is based on the standard electroweak theory, and in order to couple the monopoles to the photon and weak bosons one considers $m \gg m_{Z_0}$, m_{Z_0} being the mass of the Z_0 boson, and the monopole is taken to interact with the fundamental fields of the $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ theory before symmetry breaking, i.e., with the isoscalar field B, in the conventional notation of the standard model [32, 33]. In this way the photon, γ, and the weak boson, Z_0, have the same coupling except for an additional $\tan\theta_\mathrm{W}$, where θ_W is the Weinberg angle, for the latter. The effective description is based on the one loop approximation of the fundamental theory, and therefore the effective coupling is proportional to $g_\mathrm{eff} \sim \frac{\omega}{m} g$, where ω is an energy scale that is below the monopole production threshold, thus rendering the theory perturbative. The dynamical scheme proposed by Ginzburg and Schiller leads to effective couplings in a vector-like theory between the monopole and the photon [29, 30], given by

$$g^\gamma_\mathrm{eff} = C(J_m) g \frac{\omega}{m} = C(J_m) \frac{\omega}{2em}\,, \tag{2}$$

with $C(J_m) \sim 1$, and ω is the photon energy, m the monopole mass and N the monopole charge. The effective interaction between the monopole and the Z_0 becomes

$$g^Z_\mathrm{eff} = \tan(\theta_\mathrm{W}) g^\gamma_\mathrm{eff}\,, \tag{3}$$

where θ_W is the Weinberg angle and naturally here ω refers to the Z_0 energy. We have used the Dirac quantization condition to express the coupling in terms of the electron charge.

We study the process

$$\begin{aligned} e^+e^- &\to A \to M + A' \\ &\hookrightarrow B + C\,, \end{aligned} \tag{4}$$

shown in Fig. 1, where A, A', B and C can be γ or Z_0, in all allowed combinations, and M represents a monopolium state. We assume that the particle A' carries away the spin of the photon, and therefore M represents the lowest scalar monopolium state. We shall restrict the present calculation to photons; then the coupling arises as a consequence of the generalization of scalar electrodynamics [31].

The standard expression for the cross section in these cases results in

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = \frac{\pi}{E_e^2} G \frac{4M^2 \Gamma_{ee(A)A'} \Gamma_{BC}}{(Q^2 - M^2)^2 + M^2 \Gamma_\mathrm{M}^2}\,, \tag{5}$$

where in our case

$$G = \frac{2J_\mathrm{M} + 1}{(2s_e + 1)(2s_e + 1)} = \frac{1}{4}\,. \tag{6}$$

We have written the formula such that it looks similar to the one used in traditional decays of resonant states. Here, $x = \omega^2_{A'}/M^2$, $2E_e$ is the center of mass energy of the collision, M represents the monopolium mass, Γ_M is for the monopolium width, and Q for the monopolium four-momentum. The two remaining Γ will be defined shortly. We recall that the so called unitarity bound restricts the validity of the Ginzburg–Schiller approximation to $M < m/6$ [10, 11]. The physical situation of interest occurs when $M \ll m$ and consequently, from (2) and the fact

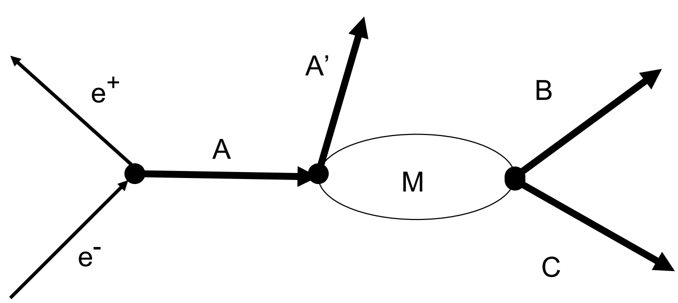

Fig. 1. Diagrammatic description of the reactions studied

[1] The Dirac quantization condition does not specify the spin of the monopoles. One could have chosen monopoles of spin 1/2. In this case, one could produce besides monopolium of spin zero, decaying into two photons, monopolium of spin 1, decaying into three photons. Since the strong coupling limit of the Dirac equation and its non-relativistic limit are technically complex [28], we have avoided in this first calculation those difficulties by limiting ourselves to bosonic monopoles. The estimates for the production will not change much as one can show in the limit in which the wave function is not relevant, namely resonance.

[2] The description in terms of quark–antiquark annihilation is straightforward, although complicated by the partonic description of the real experimental probes, which are hadrons.

that $\omega \sim M$, one gets $g_{\text{eff}} \sim g(M/m) \ll 1$, which grants validity to the perturbative approach.

We enter now the computation of the Γ. Γ_{BC} represents the width of the decay of monopolium into 2γ. The calculation, following standard field theoretic techniques of the decay of a non-relativistic bound state [34, 35], leads to

$$\Gamma_{BC} = \frac{32\pi\alpha_B\alpha_C}{m^2}|\psi_{\text{M}}(0)|^2 . \tag{7}$$

We have used the conventional approximations, namely that the particles are on shell and that in the calculation of the elementary process we neglect the binding energy. The α here correspond to the photon–monopole coupling.

The second Γ corresponds really to the calculation of the cross section for the process $M\gamma \to e^+e^-$. Following also standard techniques one arrives at

$$\sigma(M\gamma \to \gamma\gamma) = 32\pi\frac{\alpha_e\alpha_g^2}{m^2E_e^2\omega}|\Psi(0)|^2 , \tag{8}$$

where ω represents the energy of the initial photon and E_e that of the leptons in the center of mass system. From the definition of the total cross section,

$$\Gamma_{ee(A)A'} = \frac{\omega m^2\sigma(M\gamma \to \gamma\gamma)}{32\pi^2} , \tag{9}$$

which can be written for the purposes of our calculation as

$$\Gamma_{ee(A)A'} = \frac{\alpha_e\alpha_A\alpha_{A'}}{E_e^2}|\Psi(0)|^2 . \tag{10}$$

Finally (5) leads to the following differential cross section:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}w_{A'}} &= \pi^2\frac{\omega_{A'}^3\omega_B^2\omega_C^2}{\alpha^3M^4m^8E_e^2} \\ &\times \frac{|\Psi(0)|^4}{\left((2E_e-\omega_{A'})^2-\omega_{A'}^2-M^2\right)^2+M^2\Gamma_{\text{M}}^2} .\end{aligned} \tag{11}$$

In order to obtain the total cross section in the center of mass, this expression has to be integrated subject to the kinematical constraints arising from energy momentum conservation,

$$\begin{aligned}\omega_{A'}+\omega_B+\omega_C &= 2E_e , \\ \mathbf{k}_{A'}+\mathbf{k}_B+\mathbf{k}_C &= \mathbf{0} .\end{aligned} \tag{12}$$

From now on α will always denote the fine structure constant (1/137).

In order to go ahead with the calculation, one has to obtain the wave function corresponding to monopolium. This is done and analyzed in the next section.

3 Monopolium potential

Our calculation of monopolium reduces to a quantum mechanical bound state calculation, which provides us with its mass and its wave function in the relative frame. We shall use a static non-relativistic approximation, and therefore our first step is to define the potential that binds the poles to form monopolium.

We regard the monopole as possessing some spatial extension in line with the arguments of Schiff and Goebel [36, 37]. This assumption makes the potential energy of the monopole–antimonopole interaction non-singular when the relative separation goes to zero. Mathematically we describe this feature by means of an exponential cut-off in the interaction potential,

$$V(r) = -g^2\left(\frac{1-\exp(-\mu r)}{r}\right) . \tag{13}$$

We fix the cut-off parameter μ by physical arguments. Equation (13) has the following properties: for $r \to \infty$ we have

$$V(r) \to -\frac{g^2}{r} , \tag{14}$$

and for $r \to 0$

$$V(r) \to -g^2\mu + \dots \tag{15}$$

When the monopole and antimonopole are closest to each other the distance between the corresponding centers O and O' is $r_{OO'} \sim 2r_{\text{m}}$, where r_{m} is the pole radius. For our estimates $r_{\text{m}} \sim 4r_{\text{classical}}$ seems reasonable, since this choice will allow the monopolium bound state to have a very small mass for strong binding. Then

$$\mu = 2\frac{m}{g^2} . \tag{16}$$

Consequently, the effective potential finally becomes

$$V(r) = -g^2\frac{1-\exp\left(-2\frac{r}{r_{\text{classical}}}\right)}{r} . \tag{17}$$

Note that with our choice, for $r \to 0$, $V(r) \to -2m$. Thus, the mass of the bound state becomes the energy over the minimum

$$M = 2m + E_{\text{binding}} . \tag{18}$$

Summarizing, our analysis shows that the cut-off potential is quite close to the Coulomb potential as long as the monopole radius, r_{m}, is larger than the classical monopole radius $r_{\text{classical}}$. Thus, we shall use the “magnetic” Coulomb potential (Fig. 2) as our interaction in what follows. However, it is important to note, as will be shown next, that the lowest energy states of the cut-off potential correspond to excited states of the magnetic Coulomb potential (Fig. 2).

We use a non-relativistic approximation whose validity we shall shortly discuss. Solving the Schrödinger equation for monopolium we obtain its binding energy, and therefore the mass of the system is given by [38]

$$M = 2m - \left(\frac{1}{8\alpha}\right)^2\frac{m}{n^2} > 0 , \tag{19}$$

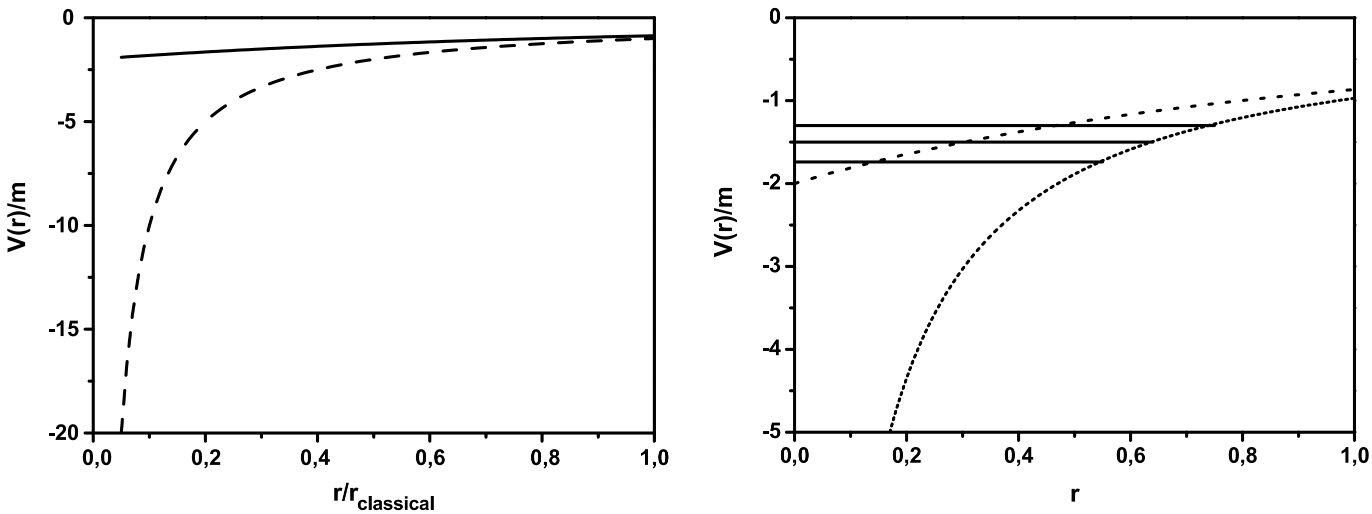

Fig. 2. The figure on the left shows the Coulomb and the Coulomb with exponential cut-off potentials. The figure on the right shows energy levels of the Coulomb potential used as energy levels of the cut-off potential. Note that the lowest energy states of the cut-off potential correspond to excited states of the Coulomb potential

where α is the fine structure constant and n is the principal quantum number. We see that we can reach zero mass for $n \sim 12$ and therefore for $n > 17$ the formula is well defined and describes all values of M:

$$0 \le M \ \le 2m\,.$$

The monopolium radius is given by

$$\frac{r_{\mathrm{M}}}{r_{\mathrm{classical}}} = 48\alpha^2 n^2\,. \tag{20}$$

Now we introduce the size parameter $\rho = r_{\mathrm{M}}/r_{\mathrm{classical}}$.

By substituting n^2 from (20) into (19), we obtain an equation for the monopolium mass as a function of its size, namely

$$M = m\left(2 - \frac{3}{4\rho}\right)\,, \tag{21}$$

which is plotted in Fig. 3. Although for low values of ρ our approximation becomes worse, we expect that the soft behavior of the wave function at the origin allows for order of magnitude estimates. This formula is extremely important in our development because it transmutates principal quantum numbers of the Coulomb potential into mass scales that are crucial to substantiate our scenario.

The perturbative expansion of Ginzburg and Schiller used in the calculation of the production process limits, due to unitarity, the maximum value of the ratio of $\frac{M}{m}$ to values $< 1/6$ [10, 11]. We incorporate this external additional requirement to the bound state calculation, which is free from it, by showing the bound in all relevant figures.

Before we continue, a discussion of the validity of the non-relativistic approximation is necessary. Let us therefore analyze how relativistic corrections will affect our calculation. As the monopoles are spin 0 bosons one may describe the system by a Schrödinger type equation and the first corrections to it do not start at order β^2 but at order β^4 (apart from merely the kinetic terms) [28]. These additional terms to order β^6 are

$$-\frac{\Delta^4}{8m^4}\,, \quad -\frac{\Delta^6}{16m^6}\,, \quad \frac{g^2}{32m^5}\left[\Delta^2, \left[\Delta^2, \frac{1}{r}\right]\right]\,. \tag{22}$$

They can be treated as perturbations to the Schrödinger equation. In the appendix we give a detailed account of the calculation of these corrections.

We show in Fig. 4 that the correction to the potential energy can be neglected safely, for an order of magnitude estimate, if the kinetic terms are fully taken into account. Moreover, we show that the minimal relativistic correction, consisting of the simplest approximation to the Klein–Gordon equation, namely in incorporating the mass [28],

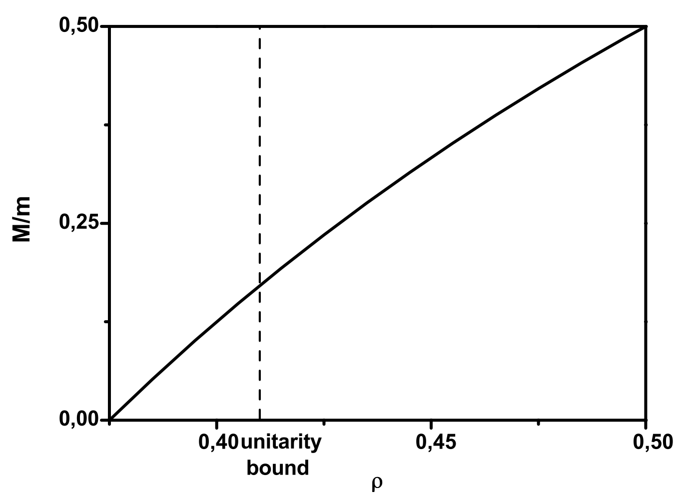

Fig. 3. Mass of the monopolium as a function of the size parameter. The unitarity bound corresponding to $\rho \sim 0.41$ is shown

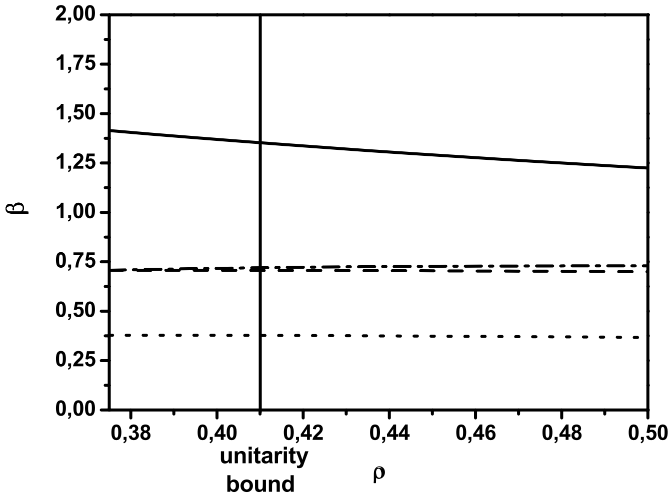

Fig. 4. Shown are the non-relativistic β (*solid*), the relativistic bosonic β_{rel} (*dashed*), the first order correction β_{rel} (*dot-dashed*) and the ratio of an upper bound to the order β^4 potential term to the binding energy to order β^4 with correct kinematics (*dotted*). The unitarity bound is also shown

leads to a velocity

$$\beta_{\mathrm{rel}}^{(1)} = \frac{\beta}{1+\frac{\beta^2}{2}}, \tag{23}$$

which is almost indistinguishable from the more complete one given by (A.6).

Furthermore, since our potential is cut off for small values of r we expect a slow down of the particles with respect to the conventional Coulomb potential and therefore the non-relativistic treatment is more accurate than in the pure Coulomb case. Moreover, the calculation for the wave function at the origin is less sensitive to the short range behavior of the potential than the velocity, which depends on the slope of the wave function.

4 Cross section estimates

We proceed to use the formalism just developed to describe the production and decay of monopolium. The analysis that follows is physically appealing because the mass of monopolium may be chosen to be small, much smaller than the monopole mass; thus detection can occur at relatively low energies. Monopolium production is accompanied by radiation, which is also described by the formalism. Furthermore the calculation is easy to perform and physically understandable.

It seems therefore safe to go ahead to calculate the monopolium decay probability as a function of ρ. The range of values of ρ is $3/8 < \rho < \infty$. Moreover,

$$n = \frac{1}{4\alpha}\sqrt{\frac{\rho}{3}},$$

and therefore, given a value of ρ, one can determine n and this fixes $|\psi(0)|^2$, which is what one needs for computing the decay probability. In summary, the calculation seems to be feasible in terms of only one mass scale, the mass of the monopole, m, and one parameter, ρ.

Let us consider the case when monopolium is produced in its ground state; its wave function will have $\ell = 0$. Consequently [38]

$$\psi_{n,0,0} = \frac{1}{a^{3/2}} N_{n,0} F_{n,0}\left(\frac{2r}{na}\right) Y_0^0(\Omega), \tag{24}$$

with

$$a = \frac{1}{m_{\mathrm{r}} e^2}, \quad N_{n,0} = \frac{2}{n^2}\sqrt{\frac{(n-1)!}{(n!)^3}},$$

where m_{r} is the reduced mass and

$$F_{n,0}(x) = e^{-1/2x} L_{n-1}^1(x);$$
$$L_{n-1}^1(x) = \sum_{s=0}^{n-1} (-1)^s \frac{(n!)^2}{(n-s-1)!(s+1)!s!} x^s.$$

We need $|\psi_{n,0,0}(0)|$. Then, taking into account that

$$\lim_{x\to 0} L_{n-1}^1(x) = nn!, \quad \lim_{x\to 0} F_{n,0}(x) = nn!,$$

one has

$$|\psi_{n,0,0}(0)| = \frac{1}{\sqrt{2\pi}(an)^{3/2}}. \tag{25}$$

The reduced mass of the monopolium system is $m/2$ and the Dirac condition (1) can be written as

$$g^2 e^2 = \frac{1}{4} \Rightarrow \alpha_g \alpha_e = \frac{1}{4},$$

and one gets

$$|\psi_{n,0,0}(0)| = \frac{1}{\sqrt{\pi}}\left(\frac{m}{8\alpha_e n}\right)^{3/2}. \tag{26}$$

Finally, one can write the wave function in terms of the variable ρ to obtain

$$|\psi_{n,0,0}(0)| = \frac{1}{\sqrt{\pi}}\left(\frac{\sqrt{3}m}{2\sqrt{\rho}}\right)^{3/2}. \tag{27}$$

This is the main ingredient to be included in the expression for the cross section that was computed before.

We are interested in spin 0 monopoles with n large and $\ell = 0$. Replacing the value of the wave function given in (27), and using the relation between M and ρ, the cross section, (11), becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_{A'}} = \frac{1}{\alpha^3 M^2 m^2 E_e^2}\left(2-\frac{M}{m}\right)^3 \times \frac{\omega_{A'}^3 \omega_B^2 \omega_C^2}{\left[\left((2E_e-\omega_{A'})^2-\omega_{A'}^2-M^2\right)^2+M^2\Gamma_{\mathrm{M}}^2\right]}. \tag{28}$$

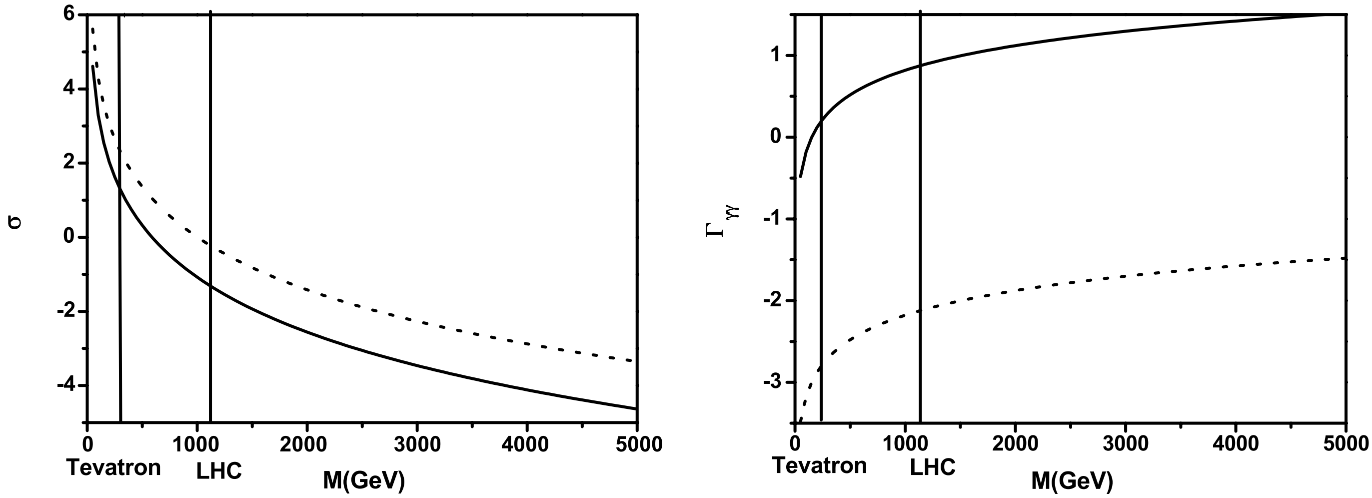

Fig. 5. Logarithmic plot of the cross section at resonance (in fb) (left) and of the monopolium width (in GeV) (right) as a function of the monopolium mass M (in GeV). The *vertical lines* represent the Tevatron ($M \sim 265$ GeV) [13] and the LHC ($M \sim 1100$ GeV) bounds. In the left plot the *full lines* have been calculated for a beam energy slightly larger than the monopolium mass, $2E_e = M + 100$ GeV, to avoid the threshold zero and maximize monopolium production. The different curves are obtained for the values of $M/m = 0.01$ (*solid*) and $M/m = 0.001$ (*dotted*)

The case that $A = Z_0$, $A' = B = \gamma$ and $C = \gamma$ also contributes to the 3γ cross section. We omit it here for simplicity, since we are just estimating the observability of the process, not its precise magnitude.

The total cross section is obtained by performing the integration

$$\sigma(\omega_{A'}, \omega_C) = \int_0^{\omega_{A'}} \mathrm{d}\omega \frac{\mathrm{d}\sigma(\omega, \omega_C)}{\mathrm{d}\omega} . \qquad (29)$$

It is evident from (28) that the cross section has a resonant structure for

$$\omega_{A'} = E_e \left(1 - \left(\frac{M}{2E_e} \right)^2 \right) .$$

In order to maximize the cross section we shall set ourselves on top of the resonant peak, and we moreover shall choose

$$\omega_B = \omega_C = \frac{E_e}{2} \left(1 + \left(\frac{M}{2E_e} \right)^2 \right) ,$$

which are the values for the energy that maximize the numerator, $\omega_B^2 \omega_C^2$, while satisfying the conservation equations (12).

We shall make a further approximation, namely that the monopolium width is dominated by the 2γ decay,

$$\begin{aligned} \Gamma_{\mathrm{M}} &\sim \Gamma_{BC}(\omega_B = \omega_C = M/2) \\ &= \frac{1}{8\alpha^2} \left(\frac{M}{m} \right)^3 \left(2 - \frac{M}{m} \right)^{3/2} M . \end{aligned} \qquad (30)$$

This width does not take into account the Z_0 decays, which will increase it. However, they will also increase the numerator. Thus, for the purposes of our calculation, this difference is not relevant at the level of order of magnitude

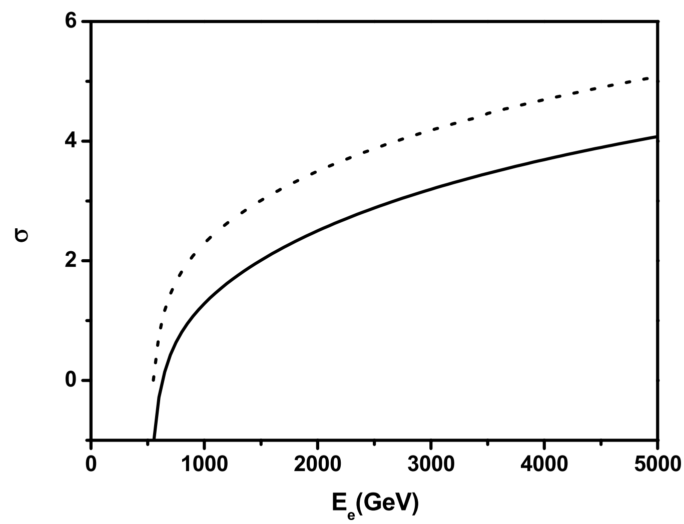

Fig. 6. Logarithmic plot of the cross section at resonance (in fb) (left) as a function of beam energy E_e for a fixed mass of the monopolium mass of $M = 1000$ GeV. The different curves are obtained for the values of $M/m = 0.01$ (*solid*) and $M/m = 0.001$ (*dotted*)

estimates. It is very important to note that in this case the cross section is independent of the monopolium wave function, since the dependence appearing in the numerator cancels exactly the dependence appearing in the calculation of the width.

The results of our calculation are shown in Figs. 5 and 6, on which we now comment.

We have two parameters in our calculation, namely the monopole mass m and the monopolium mass M. We use in the plots M and the ratio M/m. Unitarity imposes a restriction on the latter, $M/m < 0.15$, as can be seen in Fig. 3. We show data for two values of this ratio, 0.01, which serves for the purpose of developing our idea and clearly satisfies the unitarity bound, and 0.001, which could realize certain cosmological monopole scenarios. In Fig. 5 we show on the left the cross section at resonance as a function on the mass of the monopolium and for a beam energy just above the monopolium threshold, i.e. $2E_e = M + 100\,\mathrm{GeV}$. This extreme case leads to low values for the cross section due to the vicinity of the threshold zero (see for comparison the values in Fig. 6 away from threshold), but it is physically very appealing because the photon of the first vertex, A', carries almost no energy and we have a representation of the scalar monopolium decay with two photons appearing back to back with an energy of $M/2$. In this case the photon of the first vertex simply is present to carry away the spin of the intermediate photon but (almost) no energy. We see that the value of the cross section increases as M/m decreases. Thus, initially monopolium made of heavier monopoles, for a fixed mass, would be easier to see, if it were not because as shown in the right figure, their width decreases dramatically.

We include the limits of the Tevatron and LHC. Note, that both in the Tevatron and in LHC, the expected processes are the inverse of the studied here, namely $\gamma - \gamma$ fusion (and other fusion alternatives) of producing monopolium [29, 30, 39].

Let us emphasize at this point, looking at our results, the beauty of the two scale scenario. The existence of an additional scale, besides the monopole mass m, represented in our scheme by the parameter M/m, renders the present investigation exciting. If monopolium is a strongly bound system, it is its relatively low mass, M, which limits the usefulness of accelerators for monopole physics and not the mass of the monopole, conventionally assumed to be very large. If monopoles could bind to an almost zero mass bound state, we could study monopole physics at relatively low energies.

In Fig. 6 we show for a fixed monopolium mass how the peak cross section changes with the energy of the beams. We see that the threshold effect is narrow and that the cross section jumps several orders of magnitude with beam energy. Thus 3γ detection seems much more favorable than the 2γ one shown before. The signal is clear: one photon recoiling against two others, whose dynamics describes a resonant structure. Once the threshold effect disappears, the cross section is relatively flat with energy.

In Fig. 7, we fix the outcoming energy of the photon $\omega_{A'} = M/2$ and study the behavior with beam energy around the resonance. The left curve shows the differential cross section, while the right one is for the total cross section. The resonance effect is apparent and also the wave function effect. As is seen, away from resonance, the wave function effect is important and the cross section decays faster for larger monopole masses. We use for the figures $M = 1000\,\mathrm{GeV}$.

Let us summarize our findings. In our two scale scenario,

1. the differential and total cross sections have a resonant peak (see Fig. 7) determined by the monopolium mass M;
2. the order of magnitude of the cross section on resonance, which is wave function independent, rises slowly with beam energy, once we are away from the threshold, and is consistent with observability in present day machines [29, 30, 39–42] for monopolium masses of up to 2000 GeV (see Fig. 5) and monopole masses only limited by the validity of our theoretical approach;
3. a similar analysis can be carried out for γZ_0 and $2Z_0$ decays;
4. a similar analysis can be carried out for hadronic production, complicated by the inclusion of the substructure of the intervening hadrons.

5 Conclusions

We have performed an investigation looking for hints of the monopoles so far not seen. Our working assumption is

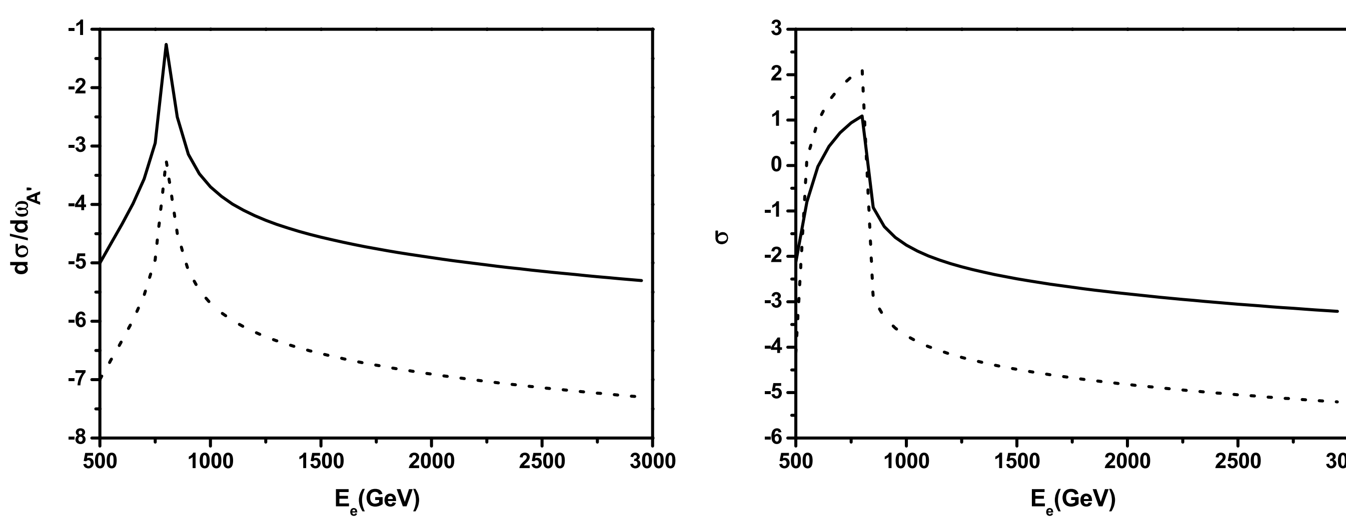

Fig. 7. Logarithmic plot of the differential in fb/GeV (left) and total cross section in fb (right) as a function of beam energy E_e for a fixed mass of the monopolium of 1000 GeV. The different curves are obtained for the values of $M/m = 0.01$ (*solid*) and $M/m = 0.001$ (*dotted*)

that monopoles appear strongly bound forming monopolium, a monopole–antimonopole bound state, due to their strong electromagnetic interaction.

We develop a scenario in which monopolium is produced and disintegrates into 2γ, γZ_0 and $2Z_0$. We give details on the structure and magnitude of the first of these processes to determine the observability. We develop a two energy scale scenario, whose

1. low scale is governed by monopolium and we consider for quantitative purposes that it could be reachable by present day machines;
2. and whose high-energy scale is governed by the monopole mass and arises through the structure of monopolium, with

$$m \gg E_e \, .$$

Under these circumstances we can estimate the cross section as a function of the monopolium mass. The monopole mass is determined by the value of the cross section and any mass is attainable, the limitations only arising from the approximations used in our model.

Since at present we cannot calculate the monopolium parameters, M and Γ_{M}, the experimental endeavor is not easy. There are however some features which might simplify the task:

1. the resonance peak of the monopolium can be found in the four exit channels 3γ, $2\gamma Z_0$, $\gamma 2Z_0$ and $3Z_0$;
2. monopolium can be produced in an excited state before it annihilates; thus the annihilation process will be accompanied by a Rydberg radiation spectrum;
3. the same processes can be studied hadronically, the only complication arising from the inclusion of the hadron sub-structure.

The calculated values for the cross sections, corresponding to reasonable monopolium mass scenarios, render our calculation interesting and this line of research worth pursuing.

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Appendix : Relativistic corrections

Let us define the relativistic factor $\beta = v/c$ through the equation

$$\beta^2 = \langle nlm_l m_s | \frac{p^2}{m^2} | nlm_l m_s \rangle \, . \tag{A.1}$$

It can easily be calculated using the exact expectation value to give [38]

$$\beta = \sqrt{\frac{3}{4\rho}} \, . \tag{A.2}$$

This result, which coincides with the semiclassical treatment and the use of Ehrenfest's theorems, namely equating the centrifugal and Coulomb forces,

$$m\frac{v^2}{r} = \frac{e^2}{r^2} \Rightarrow \frac{p^2}{2m} = \frac{1}{2}\frac{e^2}{r} \, ,$$

leads to

$$E = \frac{p^2}{2m} - \frac{e^2}{r} = \frac{p^2}{2m} - \frac{p^2}{m} = -\frac{p^2}{2m} \, .$$

This corresponds to equating the absolute values of the kinetic and the binding energies,

$$\text{kinetic energy} = |\text{binding energy}| \, , \tag{A.3}$$

which gives

$$\frac{p^2}{m} = \left(\frac{1}{8\alpha}\right)^2 \frac{m}{n^2} \, ,$$

which gives rise, using (20), to

$$\frac{p^2}{m^2} = \frac{3}{4}\frac{1}{\rho} \, .$$

Thus, the non-relativistic calculation is only truly valid for $\rho > 3/4$.

One can easily incorporate the kinetic terms in (22) to the lowest order approximation, leading to

$$m\left(\frac{\beta^2}{2} - \frac{\beta^4}{8} + \frac{\beta^6}{16} + \dots\right) \, . \tag{A.4}$$

Performing the virial theorem calculation,

$$E_{\mathrm{total}} = m\left(1 + \frac{\beta^2}{2} - \frac{\beta^4}{8} + \frac{\beta^6}{16}\right) \, , \tag{A.5}$$

and therefore the relativistic velocity turns out to be

$$\beta_{\mathrm{rel}} = \frac{\beta}{1 + \frac{\beta^2}{2} - \frac{\beta^4}{8} + \frac{\beta^6}{16}} \, . \tag{A.6}$$

The non-relativistic velocity, β, and the relativistic one, β_{rel}, are shown in Fig. 4. Finally, the correction to the potential compared to the binding energy becomes

$$\frac{\frac{g^2}{32m^4}\left\langle \left[\Delta^2, \left[\Delta^2, \frac{1}{r}\right]\right]\right\rangle}{|E_{\mathrm{binding}}|} < \frac{g^2}{32}\frac{p^3}{m^3} \approx \frac{137}{128}\beta^3_{\mathrm{rel}} \, . \tag{A.7}$$

The upper bound is also shown in Fig. 4.

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